Mathematics 433 Fall 2010 Problems to Turn in: 3

October 17

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

"The one real object of education is to have a man in the condition of continually asking questions."
-Bishop Mandell Creighton

Problems

1. Do **both** of the following.

   (a) Let $(G, \cdot)$ be a group, with multiplicative notation. Define an **opposite group** $(G, \circ)$ with law of composition $a \circ b$ as follows: The underlying set is the same as for $(G, \cdot)$, but the law of composition is the opposite; that is, define $a \circ b = b \cdot a$. Prove that this defines a group.

   (b) Prove that in any group G and for any elements $a, b \in G$, the orders of ab and ba are the same. That is, prove that the cyclic subgroups $\langle ab \rangle$ and $\langle ba \rangle$ have the same number of distinct elements.

2. Do both of the following:

   (a) Prove that if G is a group with the property that the square of every element is the identity, then G is abelian.

   (b) Let G be a finite group. Show that the number of elements x of G such that $x^3 = e$ is odd. Show that the number of elements x of G for which $x^2 \neq e$ is even.

3. Do any two of the following

   (a) Prove that every subgroup of a cyclic group is cyclic.

   (b) Describe all groups G that contain no proper subgroups.

   (c) Let $G = \langle x \rangle$ be a cyclic group of order n and let r be an integer dividing n.Say, $n = rs$. Prove that G contains exactly one subgroup of order r.